

Supplemental material for:  
Dynamic jump intensities and risk premiums:  
evidence from S&P500 returns and options

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In this supplement we provide four sets of additional results. In Section 1 we derive continuous-time limits of our models and compare them to existing models in the literature. In Section 2 we provide more detail on the risk premiums in our model. In Section 3 we present the affine component GARCH model and provide estimation results. Finally, in Section 4 we show autocorrelation plots for the models' return innovations.

## 1 Model comparison using continuous-time limits

In this section we derive the continuous-time limits for our model. Denoting the size of the time step by  $\Delta$ , our log return dynamic and the variance and jump intensity processes can be written as follows

$$R(t + \Delta) = r + (\lambda_z - 1/2)h_z(t + \Delta) + (\lambda_y - \xi)h_y(t + \Delta) + z(t + \Delta) + y(t + \Delta)$$

and

$$\begin{aligned} h_z(t + \Delta) &= w_z(\Delta) + b_z(\Delta)h_z(t) + \frac{a_z(\Delta)}{h_z(t)}(z(t) - c_z(\Delta)h_z(t))^2 + d_z(\Delta)(y(t) - e_z(\Delta))^2 \\ h_y(t + \Delta) &= w_y(\Delta) + b_y(\Delta)h_y(t) + \frac{a_y(\Delta)}{h_z(t)}(z(t) - c_y(\Delta)h_z(t))^2 + d_y(\Delta)(y(t) - e_y(\Delta))^2. \end{aligned}$$

where  $z(t+\Delta) \sim N(0, h_z(t+\Delta))$  and  $y(t+\Delta) \sim J(h_y(t+\Delta), \theta, \delta^2)$ . Note that we write the GARCH parameters as a function of the time interval  $\Delta$  in order to study their convergence as the time interval shrinks to zero.

Before deriving the limits, we need to define the variance and jump intensity as per unit time  $v_z(t) = h_z(t)/\Delta$ , and  $v_y(t) = h_y(t)/\Delta$ . Similarly, we express the normal and jump innovation  $z(t)$  and  $y(t)$  as functions of the time interval  $\Delta$  by letting

$$z(t) \equiv \sqrt{\Delta v_z(t)} \cdot z(t; \Delta) \quad \text{and} \quad y(t) = \sqrt{\Delta} \cdot y(t; \Delta), \quad (1)$$

where

$$z(t; \Delta) \sim N(0, 1) \quad \text{and} \quad y(t; \Delta) \sim J(v_y(t)\Delta, \theta(\Delta), \delta(\Delta))$$

for  $\theta(\Delta) = \theta/\sqrt{\Delta}$  and  $\delta(\Delta) = \delta/\Delta^2$ . It is easy to prove that  $y(t)$  and  $\sqrt{\Delta} \cdot y(t; \Delta)$  in (1) are identically distributed by looking at their moment generating functions. Using this parameterization, we can write the return dynamics as

$$R(t+\Delta) - r = \left( (\lambda_z - \frac{1}{2})v_z(t+\Delta) + (\lambda_y - \xi)v_y(t+\Delta) \right) \Delta + \sqrt{v_z(t+\Delta)}\sqrt{\Delta}z(t+\Delta; \Delta) + \sqrt{\Delta}y(t+\Delta; \Delta). \quad (2)$$

Similarly, the GARCH dynamics for the variance and jump intensity can be written as

$$v_z(t+\Delta) = \frac{w_z(\Delta)}{\Delta} + b_z(\Delta)v_z(t) + \frac{a_z(\Delta)}{\Delta} \left( z(t; \Delta) - c_z(\Delta)\sqrt{\Delta v_z(t)} \right)^2 + \frac{d_z(\Delta)}{\Delta} \left( \sqrt{\Delta}y(t; \Delta) - e_z(\Delta) \right)^2 \quad (3)$$

$$v_y(t+\Delta) = \frac{w_y(\Delta)}{\Delta} + b_y(\Delta)v_y(t) + \frac{a_y(\Delta)}{\Delta} \left( z(t; \Delta) - c_y(\Delta)\sqrt{\Delta v_z(t)} \right)^2 + \frac{d_y(\Delta)}{\Delta} \left( \sqrt{\Delta}y(t; \Delta) - e_y(\Delta) \right)^2 \quad (4)$$

## 1.1 Convergence of the model

There are various ways to obtain continuous-time limits as the time interval  $\Delta$  shrinks to zero. We follow the approach of Heston and Nandi (2000) to obtain the limits for the GARCH dynamics, and of Duan, Ritchken and Sun (2006) to obtain the limit for the Compound Poisson process.

First, consider the limit for the variance of the Normal innovation,  $v_z(t+\Delta)$ . We let the

GARCH parameters in (3) be a function of the time interval  $\Delta$  according to

$$\begin{aligned} w_z(\Delta) &= (\kappa_z \beta_z - \frac{1}{4} \sigma_z^2) \Delta^2; & a_z(\Delta) &= \frac{1}{4} \sigma_z^2 \Delta^2; & c_z(\Delta) &= \frac{2}{\sigma_z \Delta} - \frac{\kappa_z}{\sigma_z} \\ d_z(\Delta) &= \alpha_z \Delta; & e_z(\Delta) &= \gamma_z \sqrt{\Delta}; & b_z(\Delta) &= 0. \end{aligned}$$

Substituting the above relations into (3) and eliminating the expectations and quadratic variations that converge in probability to zero gives

$$v_z(t + \Delta) - v_z(t) = (\alpha_z \gamma_z^2 + \kappa_z (\beta_z - v_z(t))) \Delta - \sigma_z \sqrt{v_z(t)} \sqrt{\Delta} z(t; \Delta) + \alpha_z \left( \sqrt{\Delta} y(t; \Delta) \right)^2.$$

We now consider the convergence of  $\sqrt{\Delta} z(t; \Delta)$  and  $\sqrt{\Delta} y(t; \Delta)$ . Notice that as the time interval shrinks to zero we have the following convergence  $\sqrt{\Delta} z(t; \Delta) \implies \pm dW(t)$ , where  $W(t)$  is a Wiener process. Note that the sign in front of  $dW(t)$  does not impact its limit since the Wiener increment is symmetric. For the convergence of the Compound Poisson process, we use the results in Duan, Ritchken and Sun (2006)

$$\begin{aligned} \left( \sqrt{\Delta} y(t; \Delta) \right) &\implies (\delta X_{N(t)} + \theta) dN(t) \\ \left( \sqrt{\Delta} y(t; \Delta) \right)^2 &\implies (\delta X_{N(t)} + \theta)^2 dN(t), \end{aligned} \tag{5}$$

where  $dN(t)$  is a Poisson counting process with intensity  $v_y(t)$  and the  $X_i$ 's represent a sequence of independent standard normal random variables that are independent of  $W(t)$  and  $N(t)$ . Note that a more proper notation for  $v_y(t)$  is  $v_y(t^-)$ , where  $t^-$  indicates the left continuous convergence of the limit. However for simplicity, we drop the  $t^-$  notation. We further note that for small  $dt$ ,  $\delta X_{N(t)} + \theta$  is normally distributed with mean  $\theta$  and variance  $\delta^2$ . Letting  $Q(t)$  be distributed as  $N(\theta, \delta^2)$ , the variance dynamic of the GARCH model therefore converges weakly to

$$dv_z(t) = (\alpha_z \gamma_z^2 - \kappa_z (\beta_z - v_z(t))) dt + \sigma_z \sqrt{v_z(t)} dW(t) + \alpha_z Q(t)^2 dN(t)$$

Similarly, the return dynamic converges weakly to

$$d \ln S(t) = \left( r + (\lambda_z - \frac{1}{2}) v_z(t) + (\lambda_y - \xi) v_y(t) \right) dt + \sqrt{v_z(t)} dW(t) + Q(t) dN(t)$$

Next, consider the continuous-time limits of the jump intensity in (4). We let the GARCH parameters be functions of the time interval  $\Delta$  according to

$$\begin{aligned} w_y(\Delta) &= \kappa_y \beta_y \Delta^2 - \frac{1}{4\Delta} \sigma_y^2; & a_y(\Delta) &= \frac{1}{4} \sigma_y^2 \Delta; & c_y(\Delta) &= 2\rho_y \\ d_y(\Delta) &= \alpha_y \Delta; & e_y(\Delta) &= \gamma_y \sqrt{\Delta}; & b_y(\Delta) &= 1 - \kappa_y \Delta. \end{aligned}$$

Substituting these into (4) and eliminating all the expectations and quadratic variations that converge in probability to zero gives

$$\begin{aligned} v_y(t + \Delta) - v_y(t) &= (\alpha_y \gamma_y^2 + \rho_y^2 \sigma_y v_z(t) + \kappa_y (\beta_y - v_y(t))) \Delta - \rho_y \sigma_y \sqrt{v_z(t)} \sqrt{\Delta} z(t; \Delta) \\ &\quad + \alpha_y \left( \sqrt{\Delta} y(t; \Delta) \right)^2. \end{aligned}$$

Using the results for the limits of the Normal innovation that  $\sqrt{\Delta} z(t; \Delta) \implies \pm dW(t)$ , and for the Compound Poisson process in (5), we obtain the following weak convergence result for the jump intensity

$$dv_y(t) = (\alpha_y \gamma_y^2 + \rho_y^2 \sigma_y v_z(t) + \kappa_y (\beta_y - v_y(t))) dt + \rho_y \sigma_y \sqrt{v_z(t)} dW(t) + \alpha_y Q(t)^2 dN(t).$$

## 1.2 Summary

We have shown that under the parametric assumptions made the continuous-time limit of our model is given by

$$d \ln S(t) = \left( r + (\lambda_z - \frac{1}{2}) v_z(t) + (\lambda_y - \xi) v_y(t) \right) dt + \sqrt{v_z(t)} dW(t) + Q(t) dN(t) \quad (6)$$

$$dv_z(t) = (\alpha_z \gamma_z^2 + \kappa_z (\beta_z - v_z(t))) dt + \sigma_z \sqrt{v_z(t)} dW(t) + \alpha_z Q(t)^2 dN(t) \quad (7)$$

$$\begin{aligned} dv_y(t) &= (\alpha_y \gamma_y^2 + \rho_y^2 \sigma_y v_z(t) + \kappa_y (\beta_y - v_y(t))) dt \\ &\quad + \rho_y \sigma_y \sqrt{v_z(t)} dW(t) + \alpha_y Q(t)^2 dN(t) \end{aligned} \quad (8)$$

where  $W(t)$  is a Wiener process,  $N_t$  is a Poisson counting process with intensity  $v_y(t)$ , and  $Q(t) \sim N(\theta, \delta^2)$  represents the stochastic jump size that is independent of  $W(t)$  and  $N(t)$ .

We can write the above limits more compactly by using vector notation

$$\begin{aligned} d \ln S(t) &= \left( r + (\lambda_z - \frac{1}{2}) v_z(t) + (\lambda_y - \xi) v_y(t) \right) dt + \sqrt{v_z(t)} dW(t) + Q(t) dN(t) \\ \begin{pmatrix} dv_z(t) \\ dv_y(t) \end{pmatrix} &= \begin{pmatrix} \alpha_z \gamma_z^2 + \kappa_z (\beta_z - v_z(t)) \\ \alpha_y \gamma_y^2 + \rho_y^2 \sigma_y v_z(t) + \kappa_y (\beta_y - v_y(t)) \end{pmatrix} dt + \begin{bmatrix} \sigma_z & \alpha_z \\ \rho_y \sigma_y & \alpha_y \end{bmatrix} \begin{pmatrix} \sqrt{v_z(t)} dW(t) \\ Q(t)^2 dN(t) \end{pmatrix} \end{aligned}$$

## 1.3 The Santa-Clara and Yan model (2010)

The model of Santa-Clara and Yan (2010, henceforth SCY) falls in the class of quadratic models. The return dynamic follows

$$d \ln S(t) = \left( r + \phi(t) - \frac{1}{2} v_z(t) - \xi v_y(t) \right) dt + \sqrt{v_z(t)} dW_s(t) + Q(t) dN(t)$$

where  $W(t)$  is a Wiener process and  $N(t)$  is a Poisson process with intensity  $v_y(t)$ . The equity premium term in the SCY model is denoted as  $\phi(t)$ . Because the model is quadratic, the equity premium is a non-linear function of  $v_z(t)$  and  $v_y(t)$ .

The SCY model parametrizes the variance and the jump intensity as follows

$$v_z(t) = Z^2; \quad v_y(t) = Y^2$$

where  $Z$  and  $Y$  are latent state variables with the following dynamics

$$\begin{pmatrix} dZ(t) \\ dY(t) \end{pmatrix} = \begin{pmatrix} \kappa_z (\beta_z - Z(t)) \\ \kappa_y (\beta_y - Y(t)) \end{pmatrix} dt + \begin{pmatrix} \sigma_z \\ \sigma_y \end{pmatrix} \begin{pmatrix} dW_z(t) \\ dW_Y(t) \end{pmatrix}$$

The three Wiener processes in the SCY model are parameterized with the following correlation matrix

$$\Sigma = \begin{pmatrix} 1 & \rho_{sz} & \rho_{sy} \\ \rho_{sz} & 1 & \rho_{yz} \\ \rho_{sy} & \rho_{yz} & 1 \end{pmatrix}.$$

## 1.4 Model comparison

When comparing the continuous-time limit of our model to the SCY model, the similarity is that they both allow for dynamic volatility and dynamic jump intensity. As far as modeling differences are concerned, our model has both advantages and disadvantages compared to the SCY model.

The quadratic nature of the SCY model is an important advantage. Our model falls in the affine class, and in that sense it is closer to the class of continuous-time affine SVJ models.

On the other hand, the SCY model—as is standard—does not allow for jumps in volatility and jumps in the jump intensity whereas our model does. If we shut down the jumps in volatility and jumps in jump intensity in our model, that is we set,  $\alpha_z = 0$  and  $\alpha_y = 0$  in (7) and (8), then our model falls under the affine jump-diffusion class

$$\begin{aligned} d \ln S(t) &= \left( r + (\lambda_z - \frac{1}{2})v_z(t) + (\lambda_y - \xi)v_y(t) \right) dt + \sqrt{v_z(t)}dW(t) + Q(t)dN(t) \\ \begin{pmatrix} dv_z(t) \\ dv_y(t) \end{pmatrix} &= \begin{pmatrix} \kappa_z (\beta_z - v_z(t)) \\ \rho_y^2 \sigma_y v_z(t) + \kappa_y (\beta_y - v_y(t)) \end{pmatrix} dt + \begin{pmatrix} \sigma_z \\ \rho_y \sigma_y \end{pmatrix} \sqrt{v_z(t)}dW(t). \end{aligned}$$

This restricted version of our model is similar to the affine jump-diffusion models studied in several papers in the continuous-time literature, but it does not nest the quadratic jump-diffusion model of SCY.

## 2 Risk premium structure: comparison with existing studies

The framework in this paper leads to a simple affine specification of the equity premium in the variance of the normal innovation and the jump intensity. That is,

$$\log E_t [\exp (R_{t+1})] = \lambda_z h_{z,t+1} + \lambda_y h_{y,t+1}$$

where  $\lambda_z h_{z,t+1}$  and  $\lambda_y h_{y,t+1}$  represent the normal risk premium and the jump risk premium, respectively. We now show that the resulting decomposition of the equity premium is consistent with existing studies such as Pan (2002), Broadie, Chernov and Johannes (2007) and Liu, Pan and Wang (2005).

**Lemma 1** *Using the EMM in Proposition 1 and the risk-neutral dynamics in Proposition 2, the conditional equity premium for the return dynamic in (2.1) can be written as*

$$\lambda_z h_{z,t+1} + \xi h_{y,t+1} - \xi^* h_{y,t+1}^* \tag{9}$$

where  $\xi^* h_{y,t+1}^* \equiv \left( e^{\frac{\delta^2}{2} + \theta^*} - 1 \right) h_{y,t+1}^*$  and  $\xi h_{y,t+1} \equiv \left( e^{\frac{\delta^2}{2} + \theta} - 1 \right) h_{y,t+1}$  are the compensators to the jump component under the risk-neutral and physical measure respectively.

**Proof.** Taking the logarithm of  $E_t \left[ \frac{S_{t+1}}{S_t} \right] = e^{r + \lambda_z h_{z,t+1} + \lambda_y h_{y,t+1}}$  yields the conditional equity premium  $\lambda_z h_{z,t+1} + \lambda_y h_{y,t+1}$ . We are left to show that  $\lambda_y h_{y,t+1} = \xi h_{y,t+1} - \xi^* h_{y,t+1}^*$ . Using the EMM condition

$$\lambda_y - \left( e^{\frac{\delta^2}{2} + \theta} - 1 \right) - e^{\frac{\Lambda_y^2 \delta^2}{2} + \Lambda_y \theta} \left( 1 - e^{(\frac{1}{2} + \Lambda_y) \delta^2 + \theta} \right) = 0,$$

we can write

$$\begin{aligned} \lambda_y h_{y,t+1} &= \left( e^{\frac{\delta^2}{2} + \theta} - 1 \right) h_{y,t+1} - \left( 1 - e^{(\frac{1}{2} + \Lambda_y) \delta^2 + \theta} \right) h_{y,t+1} e^{\frac{\Lambda_y^2 \delta^2}{2} + \Lambda_y \theta} \\ &= \xi h_{y,t+1} - \xi^* h_{y,t+1}^* \end{aligned}$$

where the second line in the above equation follows from

$$h_{y,t+1}^* = h_{y,t+1} \exp \left( \frac{\Lambda_y^2 \delta^2}{2} + \Lambda_y \theta \right) \text{ and } \theta^* = \theta + \Lambda_y \delta^2 \tag{10}$$

which was shown in Proposition 2 ■

Using an equilibrium model with a representative agent, Liu, Pan and Wang (2005) also obtain expression (9) for the equity premium. They also refer to  $\lambda_z h_{z,t+1}$  as the normal risk

premium and  $\lambda_y h_{y,t+1} = \xi h_{y,t+1} - \xi^* h_{y,t+1}^*$  as the jump risk premium.

### 3 The affine component GARCH model

Christoffersen, Jacobs, Ornathanalai and Wang (2008) develop an affine component GARCH model, and demonstrate its superior option pricing performance relative to the benchmark affine Heston-Nandi GARCH(1,1) process. The richer volatility dynamics enable the component model to capture patterns in long-maturity as well as in short-maturity options. The return dynamic of the component GARCH is identical to that of the GARCH(1,1) model. It is given by

$$R_{t+1} \equiv \ln(S_{t+1}/S_t) = r + (\lambda - 1/2) h_{t+1} + z_{t+1} \quad (11)$$

where  $z_{t+1} \sim N(0, h_{t+1})$ . The variance dynamic of the component GARCH model however consists of two components: the short-run component  $h_{t+1}$ , and the long-run component  $q_{t+1}$

$$\begin{aligned} h_{t+1} &= q_{t+1} + \beta (h_t - q_t) + \frac{\alpha}{h_t} ((z_t^2 - h_t) - 2\gamma_1 z_t h_t) \\ q_{t+1} &= \omega + \rho q_t + \frac{\varphi}{h_t} ((z_t^2 - h_t) - 2\gamma_2 z_t h_t). \end{aligned} \quad (12)$$

To price options, we need the risk-neutralized version of the component GARCH model. The risk-neutral component GARCH dynamic is given by

$$\begin{aligned} R_{t+1} &\equiv \ln(S_{t+1}/S_t) = r + \left(\lambda - \frac{1}{2}\right) h_{t+1} + z_{t+1} \\ h_{t+1} &= q_{t+1} + \beta^* (h_t - q_t) + \frac{\alpha^*}{h_t} ((z_t^2 - h_t) - 2\gamma_1^* z_t h_t) \quad \text{and} \\ q_{t+1} &= \omega + \rho^* q_t + \frac{\varphi}{h_t} ((z_t^2 - h_t) - 2\gamma_2^* z_t h_t), \end{aligned} \quad (13)$$

where the risk-neutral parameters are defined as

$$\begin{aligned} \beta^* &= \beta + \alpha (\gamma_1^{*2} - \gamma_1^2) + \varphi (\gamma_2^{*2} - \gamma_2^2), \\ \rho^* &= \rho + \alpha (\gamma_1^{*2} - \gamma_1^2) + \varphi (\gamma_2^{*2} - \gamma_2^2), \text{ and} \\ \gamma_i^* &= \gamma_i + \lambda, \quad \text{for } i = 1, 2. \end{aligned}$$

We have estimated the component GARCH using the data set used in our paper. Table S1 contains the estimation results. Panel A shows the MLE estimates on returns only, and Panel B shows the MLE estimates obtained using returns and options jointly.

Comparing the Option IVRMSE of 5.15% in Panel A with the results in Table 1 of the paper, we see that the component GARCH model outperforms the DVCJ and CVDJ models,

but it is outperformed by the DVDJ and DVSDJ models. The component GARCH model in which both components are Gaussian is thus better than the single factor DVCJ and CVDJ models but worse than the two-factor DVCJ and CVDJ models. The component GARCH model also performs better than the two-factor Gaussian model in Table 2.

For the case of the joint estimation, the total log-likelihood for the component GARCH model in Panel B of Table S1 is 38,277, which is better than the GARCH and two-factor GARCH model in Table 6, but worse than the three jump models, DVCJ, DVDJ and DVSDJ. Therefore, the new jump models perform well relative to the component GARCH model in this case too.

## 4 Autocorrelations of returns and innovations

In this section we present the autocorrelations of the return data, and compare them to the autocorrelations from the model innovations. The return specification is

$$R_{t+1} \equiv r + \left(\lambda_z - \frac{1}{2}\right) h_{z,t+1} + (\lambda_y - \xi) h_{y,t+1} + z_{t+1} + y_{t+1}. \quad (14)$$

Figure S1 plots the autocorrelation function of daily S&P500 return,  $R_{t+1}$ , for 1962-2009, along with the autocorrelation of the daily total residual from our models, defined as

$$z_{t+1} + y_{t+1} = R_{t+1} - r - \left(\lambda_z - \frac{1}{2}\right) h_{z,t+1} - (\lambda_y - \xi) h_{y,t+1}. \quad (15)$$

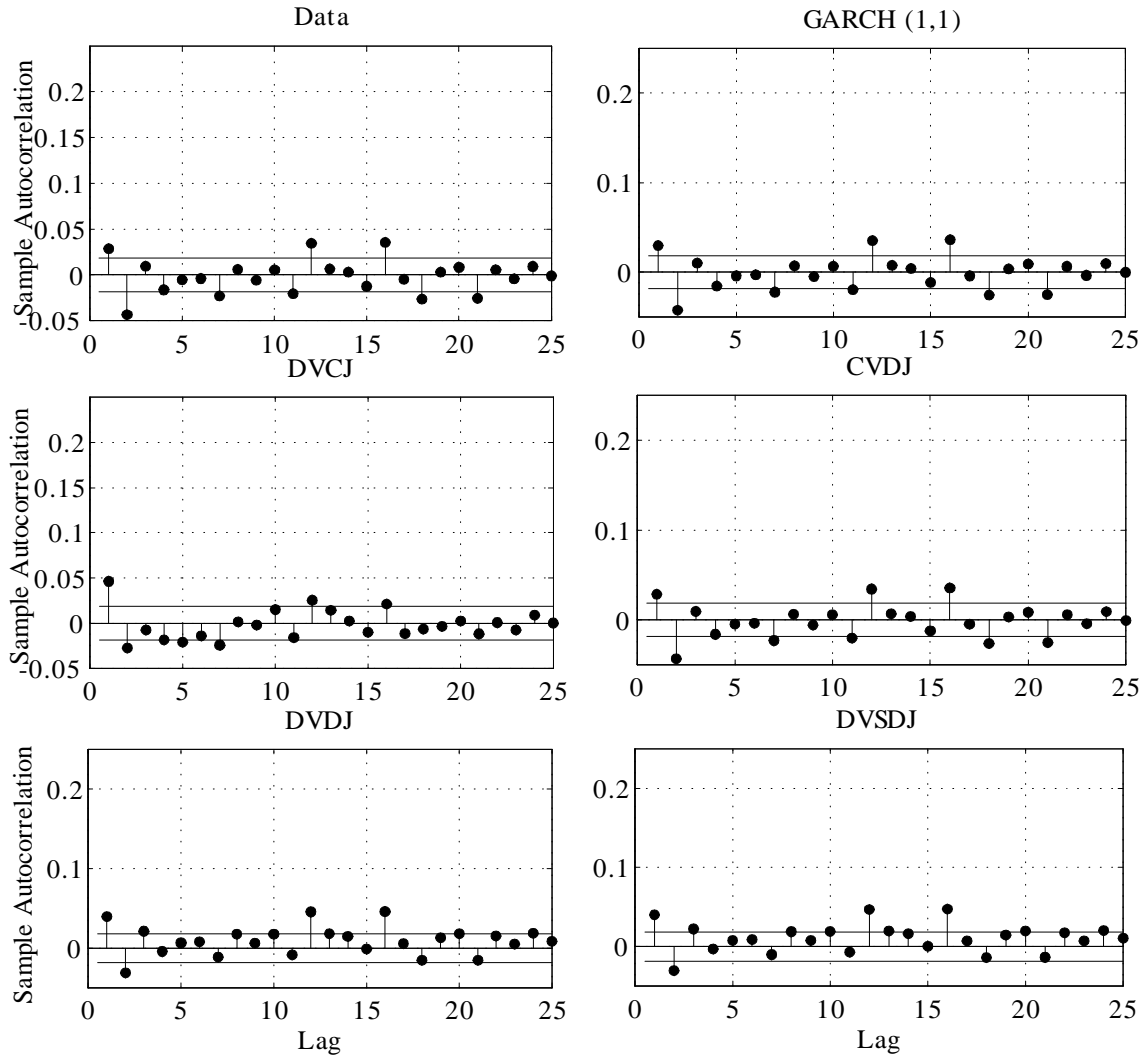
Figure S1 also shows two-standard-deviation Bartlett bands.

The top left panel in Figure S1 shows that the daily S&P500 returns display no systematic time-series patterns. The first autocorrelation is positive, perhaps capturing illiquidity in the cash index that we use. The other five panels in Figure S1 show that—not surprisingly—the residuals in the models we estimate largely retain the autocorrelation patterns from the raw returns.

## References

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Figure S1: Autocorrelation functions for S&P500 returns and model residuals. 1962-2009.



Notes to Figure: We plot the autocorrelation function (ACF) for the daily S&P500 return,  $R_{t+1}$  in the top left panel and the ACF of the model residuals,  $z_{t+1} + y_{t+1}$  in the other five panels. We use daily returns for 1962-2009.

**Table S1: MLE and joint MLE with component GARCH model****Panel A: Returns MLE (1969-2009)****Panel B: Joint options & returns MLE (1995-2009)**

<b>Parameters</b>		<b>Parameters</b>	
$\lambda$	1.60E+00 (9.12E-01)	$\lambda$	1.20E+00 (5.35E-03)
$\omega$	8.30E-07 (6.65E-08)	$\omega$	1.41E-06 (6.82E-12)
$\beta$	7.50E-01 (2.08E-02)	$\beta$	7.34E-01 (8.10E-08)
$\alpha$	1.39E-06 (2.36E-07)	$\alpha$	5.33E-06 (7.16E-08)
$\gamma_1$	4.53E+02 (7.68E+01)	$\gamma_1$	2.66E+02 (3.36E+00)
$\gamma_2$	83.2455 (7.64E+00)	$\gamma_2$	101.9434 (5.07E+00)
$\phi$	2.01E-06 (1.12E-07)	$\phi$	3.65E-06 (4.47E-08)
$\rho$	9.91E-01 (8.00E-04)	$\rho$	9.86E-01 1.95E-04
<b>Properties</b>		<b>Properties</b>	
Persistence	0.9977	Long-run risk premium	4.60%
Percent of annual variance	100%	Average annual volatility	19.61%
Average annual volatility	14.97%	Percent of annual variance	100%
Log-likelihood	40,457	Total log-likelihood	38,277
Option IVRMSE	5.15	Options log-likelihood	4,492
		Return log-likelihood	33,784

Notes to Table: We estimate a component GARCH model (see Christoffersen et al. (2008)) using MLE on returns and joint MLE on Options and Returns. Panel A reports MLE on daily S&P 500 daily returns from June 1969 to December 2009. Panel B present results jointly fitting daily S&P500 returns and weekly out-of-the money S&P 500 options with maturity of 250 days or less using data from January 1996 to October 2009. Under Properties we report relevant characteristics implied by the model from each estimation methodology.